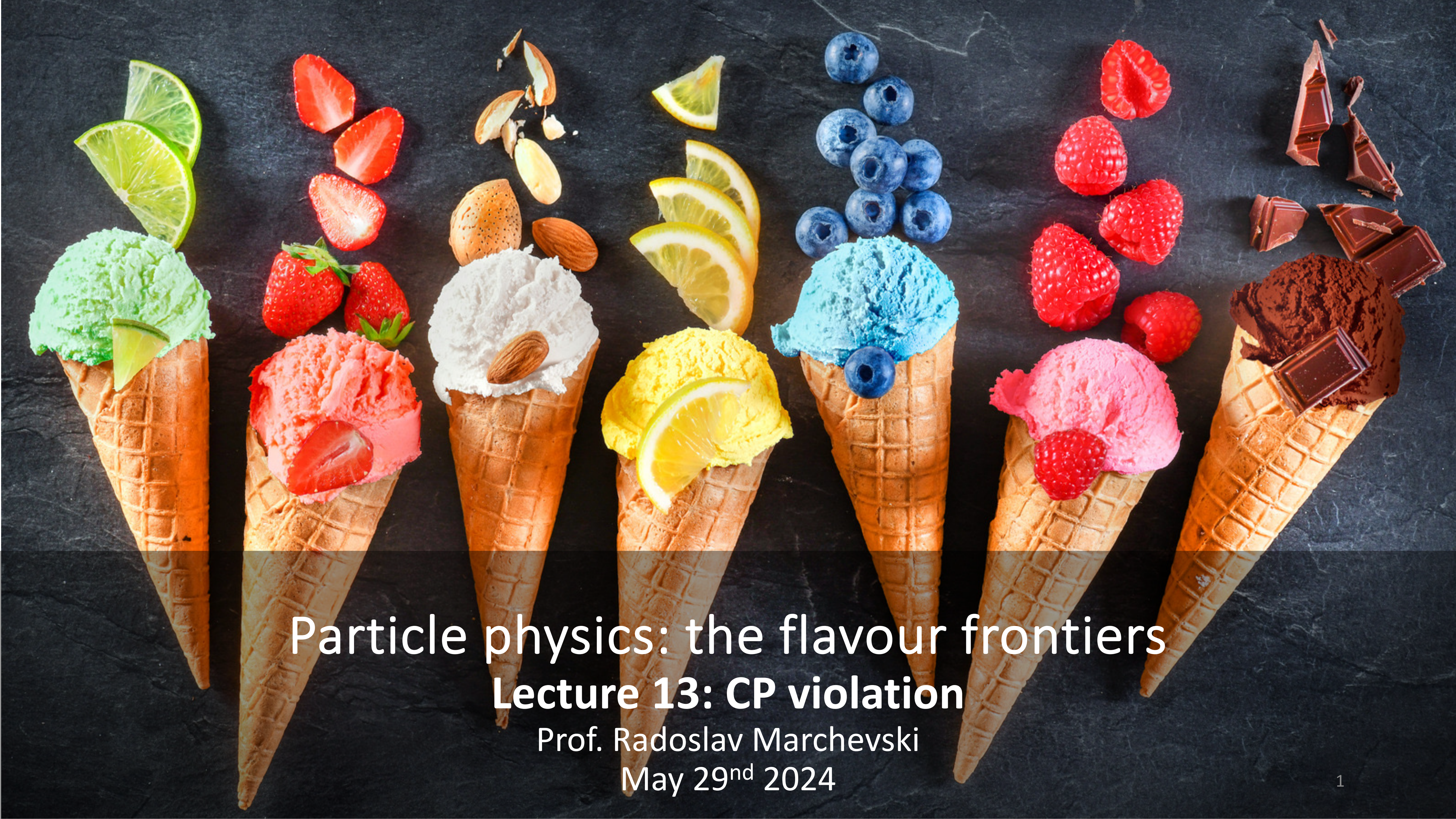

Particle physics: the flavour frontiers

Lecture 13: CP violation

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May 29nd 2024

Short recap and today's learning targets

<u>Last time we discussed</u>

- Phenomenology of neutral meson mixing and oscillation in the Standard Model
- Differences between the mixing parameters of different neutral mesons and its experimental implications (mass differences, width differences, time-dependent behaviour)
- Flavour tagging and how it can be used to experimentally measure flavour mixing and oscillation

<u>Today you will …</u>

- learn about the different types of CP-violation in the Standard Model
- explore the phenomenology of CP-violation in charged and neutral meson decays (kaons, B mesons)
- learn about the experimental measurements of CP-violation in the different meson systems

CP violation

- *CP* asymmetries arise when two processes related by *CP* conjugation differ in their rates
- *CP* violation is related to a phase in the Lagrangian $\Longrightarrow$ all *CP* asymmetries must arise from interference effects

$$x \equiv \frac{\Delta m}{\Gamma}, \qquad y \equiv \frac{\Delta\Gamma}{2\Gamma}, \qquad \lambda_f \equiv \frac{q}{p}\frac{\bar{A}_f}{A_f}$$

- Full time evolution formula

$$2\hat{\Gamma}[P^0(t) \to f] = \left(1 + \left|\lambda_f\right|^2\right)\cosh(y\Gamma t) \;+ \left(1 - \left|\lambda_f\right|^2\right)\cos(x\Gamma t) \;+ 2\mathcal{Re}\left(\lambda_f\right)\sinh(y\Gamma t) \;- 2\mathcal{Im}\left(\lambda_f\right)\sin(x\Gamma t)$$

$$2\hat{\Gamma}[\bar{P}^0(t) \to f] = \left(1 + \left|\lambda_f\right|^{-2}\right)\cosh(y\Gamma t) + \left(1 - \left|\lambda_f\right|^{-2}\right)\cos(x\Gamma t) + 2\mathcal{Re}\left(\lambda_f^{-1}\right)\sinh(y\Gamma t) - 2\mathcal{Im}\left(\lambda_f^{-1}\right)\sin(x\Gamma t)$$

- Let's take B-meson decays as an example

A_f: $B^0 \to f$ amplitude

$\bar{A}_{\bar{f}}$: $\bar{B}^0 \to \bar{f}$ amplitude of the *CP*-conjugated process

CP violation: phases

- Two types of phases appear in the decay amplitude: *CP-odd* and *CP-even* phases
- *CP-odd phases:*
  - complex parameter that changes sign under *CP* transformation between A_f and $\bar{A}_{\bar{f}}$
  - linked to $W^{\pm}$ boson interactions $\Longrightarrow$ known as weak phases
- *CP-even phases:*
  - phases can appear in decay amplitudes even if the Lagrangian parameters are all real
  - contributions from intermediate on-shell states
  - do **NOT** change sign under *CP* transformation between A_f and $\bar{A}_{\bar{f}}$
  - in meson decays the intermediate states are typically hadronic states with the same flavour quantum numbers driven by the strong interactions $\Longrightarrow$ known as strong phases

CP violation: amplitudes

- It's useful to factorise an amplitude in three parts
  - magnitude a_i
  - weak phase ϕ_i
  - strong phase δ_i
- If there are two such contributions to an amplitude we can write

$$A_f = a_1 e^{i(\delta_1+\phi_1)} + a_2 e^{i(\delta_2+\phi_2)} \qquad \bar{A}_{\bar{f}} = a_1 e^{i(\delta_1-\phi_1)} + a_2 e^{i(\delta_2-\phi_2)}$$

- We always choose $a_1 > a_2$

$$r_f = \frac{a_2}{a_1}, \qquad \phi_f = \phi_2 - \phi_1, \qquad \delta_f = \delta_2 - \delta_1$$

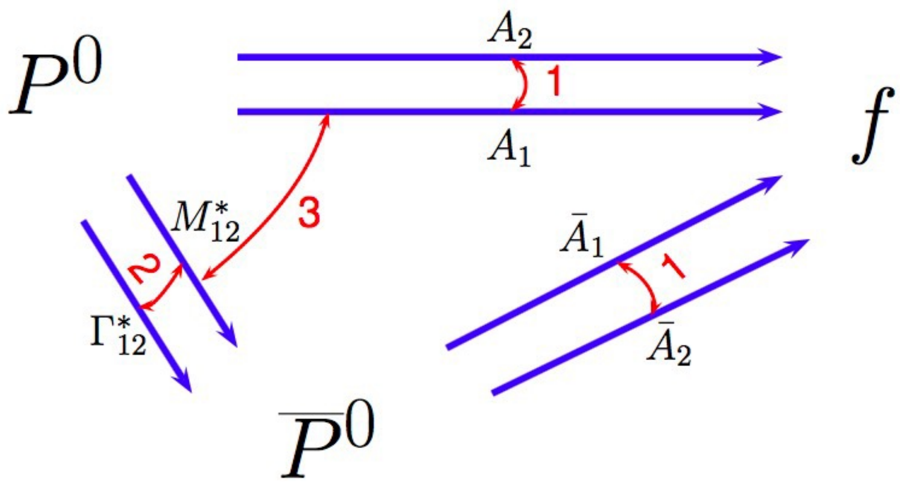
- For neutral meson mixing we can write

$$M_{B\bar{B}} = |M_{B\bar{B}}| e^{i\phi_M}, \qquad \Gamma_{B\bar{B}} = |\Gamma_{B\bar{B}}| e^{i\phi_\Gamma}$$

$$\theta_B = \arg\left(M_{B\bar{B}} \Gamma^*_{B\bar{B}}\right) = \phi_M - \phi_\Gamma$$

Types of *CP* violation

- Each phase is convention-dependent but ϕ_f, δ_f and θ_B are physical
- Phenomenology of *CP* violation is very rich in neutral meson decays: mixing can contribute to the interference
- Three types of *CP* violation mechanisms depending on which amplitudes interfere
  - *In decay:* interference between two decay amplitudes
  - *In mixing:* interference between absorbtive (on-shell intermediate states) and dispersive (off-shell intermediate state) mixing amplitudes
  - *In interference between decays with and without mixing:* interference between direct decay and first-mix-then-decay amplitude



CP violation

- *CP* violation in the kaon system is somewhat different
- Lifetimes of the two neutral kaons are very different $\tau_{K_L}/\tau_{K_S} \approx 550$
- As a consequence, it is useful to identify the mass eigenstates rather than the flavour-tagged decays
- We define

$$\epsilon_f = \frac{1-\lambda_f}{1+\lambda_f}, \qquad \lambda_f = \frac{1-\epsilon_f}{1+\epsilon_f}$$

- Historically *CP* violation was first observed in the $K_L \to \pi^+\pi^-$ decay and we denote $\epsilon_{\pi^+\pi^-} = \epsilon_K$
- Neglecting direct *CP* violation in kaons $\left(A_f/\bar{A}_f - 1 \ll |q/p| - 1\right)$

$$\epsilon_f = \frac{1-q/p}{1+q/p}, \qquad |\lambda_f| = \left|\frac{q}{p}\right|$$

CP violation in decay

$$\frac{|A_f|}{|\bar{A}_{\bar{f}}|} \neq 1$$

- In charged particle decays this is the only possible contribution to the *CP* asymmetry:

$$\mathcal{A}_f \equiv \frac{\Gamma(B^- \to f^-) - \Gamma(B^+ \to f^+)}{\Gamma(B^- \to f^-) + \Gamma(B^+ \to f^+)} = \frac{|\bar{A}_{f^-}/A_{f^+}|^2 - 1}{|\bar{A}_{f^-}/A_{f^+}|^2 + 1}$$

- Using the equation from slide 5 we obtain for $r_f \ll 1$

$$\mathcal{A}_f = 2 r_f \sin\phi_f \sin\delta_f$$

- We need two decay amplitudes $(r_f \neq 0)$ with different weak phases $(\phi_f \neq 0, \pi)$ and strong phases $(\delta_f \neq 0, \pi)$

CP violation in decay: comments

$$\mathcal{A}_f = 2r_f \sin\phi_f \sin\delta_f$$

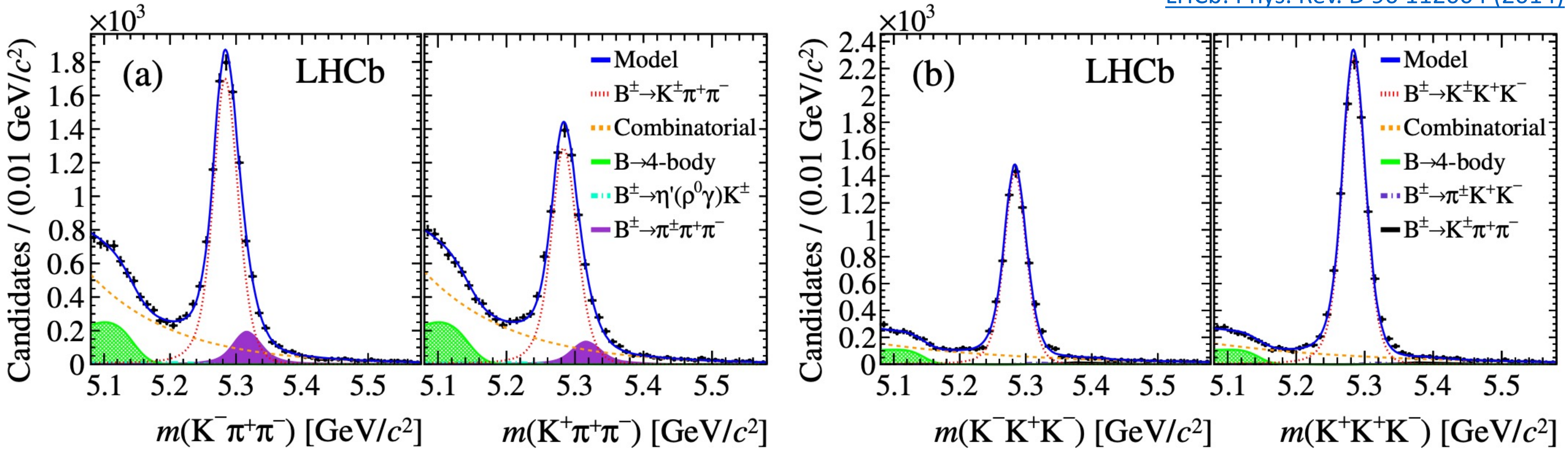
- To have large *CP* asymmetry we need each of the three factors not to be small
- Similar expression holds for the contribution of *CP* violation in decay in neutral mesons decays but with additional contributions from mixing
- Another complication in neutral meson decays is that it is not always possible to tell the floavour of the decaying meson (e.g. if it's a B^0 or $\bar{B}^0$) which can be a problem or an advantage
- In general, strong phase is not calculable since it is related to QCD
  - not a problem if the aim is to demonstrate *CP* violation
  - problem if we want to extract the weak phase ϕ_f
  - in some cases, the strong phase can be measured experimentally, eliminating the source of theoretical unceretainty

CP violation in decay: measurement

- *CP* asymmetries in charged B mesons has been observed in several decay modes
- *Example*: charmless three-body decay modes $B^{\pm} \to K^{\pm}\pi^{+}\pi^{-}, B^{\pm} \to K^{\pm}K^{+}K^{-}, B^{\pm} \to \pi^{\pm}K^{+}K^{-}, B^{\pm} \to \pi^{\pm}\pi^{+}\pi^{-}$ measured by LHCb

$$\mathcal{A}_f \equiv \frac{\Gamma(B^- \to f^-) - \Gamma(B^+ \to f^+)}{\Gamma(B^- \to f^-) + \Gamma(B^+ \to f^+)}$$

LHCb: Phys. Rev. D 90 112004 (2014)



CP violation in mixing

$$\left|\frac{q}{p}\right| \neq 1$$

- In decays of neutral mesons into favour-specific final states ($\bar{A}_f = 0$, and consequently $\lambda_f = 0$)
- In semileptonic neutral meson decays, this is the only source of CP violation

$$\mathcal{A}_{\mathrm{SL}}(t) \equiv \frac{\hat{\Gamma}\left(\overline{B}^0(t) \to l^+ X\right) - \hat{\Gamma}\left(B^0(t) \to l^- X\right)}{\hat{\Gamma}\left(\overline{B}^0(t) \to l^+ X\right) + \hat{\Gamma}\left(B^0(t) \to l^- X\right)} = \frac{1 - |q/p|^4}{1 + |q/p|^4}$$

- Using

$$\left(\frac{q}{p}\right)^2 = \frac{M_{P\bar{P}}^* - (i/2)\Gamma_{P\bar{P}}^*}{M_{P\bar{P}} - (i/2)\Gamma_{P\bar{P}}},$$

- We obtain for $\left|\Gamma_{B\bar{B}}/M_{P\bar{P}}\right| \ll 1$

$$\mathcal{A}_{\mathrm{SL}}(t) = -\left|\Gamma_{B\bar{B}}/M_{P\bar{P}}\right| \sin(\phi_M - \phi_\Gamma)$$

CP violation in mixing

$$\mathcal{A}_{\mathrm{SL}} = -\left|\Gamma_{B\bar{B}}/M_{P\bar{P}}\right| \sin(\phi_M - \phi_\Gamma)$$

- The $\mathcal{A}_{\mathrm{SL}}(t)$ quantity which is an asymmetry of time-dependent decay rates, is actually time independent
- The calculation of $\left|\Gamma_{B\bar{B}}/M_{P\bar{P}}\right|$ is difficult because it depends on low-energy QCD
- The extraction of the value of the *CP* violating phase $\phi_M - \phi_\Gamma$ from a measurement of $\mathcal{A}_{\mathrm{SL}}$ involves, in general, large hadronic uncertainties
- CP violation in mixing is measured via semileptonic asymmetry

$$\delta_L \equiv \frac{\Gamma(K_L \to l^+\nu_l\pi^-) - \Gamma(K_L \to l^-\nu_l\pi^+)}{\Gamma(K_L \to l^+\nu_l\pi^-) + \Gamma(K_L \to l^-\nu_l\pi^+)} = \frac{1-|q/p|^2}{1+|q/p|^2} \approx 2\mathcal{R}e(\epsilon_K)$$

- Different from that in the B system because the decaying meson is the mass eigenstate rather than flavour eigenstate and therefore has different dependence on q/p

CP violation in interference of decays with and without mixing

$$\mathcal{Im}(\lambda_f) \neq 0$$

- CP asymmetry in decays into final CP eigenstates
- Situation relevant in many cases is when one can neglect the effects of CP violation in decay and in mixing

$$\left|\bar{A}_{f_{CP}}/A_{f_{CP}}\right| \approx 1 \qquad |q/p| \approx 1 \qquad \left|\lambda_{f_{CP}}\right| = 1$$

- If we further consider the case where we can neglect y ($y \ll 1$) then

$$\mathcal{A}_{f_{CP}}(t) \equiv \frac{\Gamma\left(\bar{B}^0(t) \to f_{CP}\right) - \Gamma\left(B^0(t) \to f_{CP}\right)}{\Gamma\left(\bar{B}^0(t) \to f_{CP}\right) + \Gamma\left(B^0(t) \to f_{CP}\right)} = \mathcal{Im}\left(\lambda_{f_{CP}}\right)\sin(\Delta m_B t)$$

- These approximations are valid in cases where $\left|\Gamma_{B\bar{B}}/M_{P\bar{P}}\right| \ll 1$ and $a_2 \ll a_1$ which lead to

$$\frac{q}{p} = \frac{M^*_{B\bar{B}}}{\left|M^*_{B\bar{B}}\right|} = e^{-i\phi_M}, \qquad \frac{\bar{A}_{f_{CP}}}{A_{f_{CP}}} = e^{-2i\phi_A}$$

ϕ_1 (associated with a_1)

CP violation in interference of decays with and without mixing

$$\mathcal{Im}(\lambda_f) \neq 0$$

$$\frac{q}{p} = \frac{M_{B\bar{B}}^*}{|M_{B\bar{B}}^*|} = e^{-i\phi_M}, \qquad \frac{\bar{A}_{f_{CP}}}{A_{f_{CP}}} = e^{-2i\phi_A}$$

ϕ_1 (associated with a_1)

$$\mathcal{Im}(\lambda_{f_{CP}}) = \mathcal{Im}\left(\frac{M_{B\bar{B}}^*}{|M_{B\bar{B}}^*|}\frac{\bar{A}_{f_{CP}}}{A_{f_{CP}}}\right) = -\sin(\phi_M + 2\phi_A)$$

- Measurement of a *CP* asymmetry in a process where these approximations are valid provides a direct probe of the weak phase between the mixing amplitude and the decay amplitude
- For the decays where we measure decays of the K_L and K_S mass eigenstates into final *CP*-even eigenstates we obtain

$$\mathcal{A}_{f_{CP}}^{\mathrm{mass}} \equiv \frac{\Gamma(K_L \to f_{CP})}{\Gamma(K_S \to f_{CP})} = \left|\frac{1-\lambda_{f_{CP}}}{1+\lambda_{f_{CP}}}\right|^2 = |\epsilon_{f_{CP}}|^2$$

$$f_{CP} = \pi^+\pi^- \Longrightarrow \mathcal{A}_{\pi^+\pi^-}^{\mathrm{mass}} = |\epsilon_K|^2$$

The neutral kaon system

- $K^0 - \bar{K}^0$: (theory and) experiments → $1 - |q/p| \approx 7\times10^{-3}$ (**small CP violation in mixing**)
- Final state $f = \pi\pi$
  - Kinematical allowed phase space **much larger** than **any other** decay channel
  - CP eigenstate (eigenvalue +1)
  - $|A_{\pi\pi}| \approx |\bar{A}_{\pi\pi}|$ (**very small direct CP-violation** → experimental and theoretical fact)
  - $\lambda_{\pi\pi}$ almost real (**small interference CP-violation** → experimental and theoretical fact)
- Small CP violation in mixing → $|P_L\rangle$ and $|P_H\rangle$ ~ CP eigenstates with eigenvalues +1 and -1
- $P_H \to \pi\pi$ very much suppressed than $P_L \to \pi\pi \implies P_L$ decays much faster than P_H
- $|P_L\rangle$ and $|P_H\rangle$ renamed $|K_S\rangle$ (short) and $|K_L\rangle$ (long)
  - $\Delta\Gamma = \Gamma_L - \Gamma_S \approx -\Gamma_S$, $\Gamma = (\Gamma_L + \Gamma_S)/2 \approx \Gamma_S/2$
  - $|K_L\rangle = p|K^0\rangle + q|\bar{K}^0\rangle \approx 2^{-1/2}(|K^0\rangle - |\bar{K}^0\rangle)$, $|K_S\rangle = p|K^0\rangle - q|\bar{K}^0\rangle \approx 2^{-1/2}(|K^0\rangle + |\bar{K}^0\rangle)$

The neutral kaon system

- Time evolution starting from pure $|K^0\rangle$ ($\sim$ from pure $|\bar{K}^0\rangle$): **hyp.** CP $\sim$ conserved, decay in $\pi\pi$ only

$$H = \left(1 + \left|\lambda_f\right|^2\right) \cosh\frac{\Delta\Gamma\, t}{2} - 2\mathcal{R}e\lambda_f \sinh\frac{\Delta\Gamma\, t}{2} \approx 2e^{-\frac{\Gamma_S}{2}t} \qquad I = \left(1 - \left|\lambda_f\right|^2\right)\cos(\Delta m\, t) + 2\mathcal{I}m\lambda_f \sin(\Delta m\, t) \approx 0$$

$$\Gamma[K^0(t) \to \pi\pi] = \frac{1}{2}|A_{\pi\pi}|^2 e^{-\Gamma t}(H + I) \sim |A_{\pi\pi}|^2 e^{-\Gamma_S t} \approx \frac{1}{2}|\langle\pi\pi|\mathcal{H}|K_S\rangle|^2 e^{-\Gamma_S t}$$

$\Rightarrow$ **The kaon decays as $\boldsymbol{K_S}$**

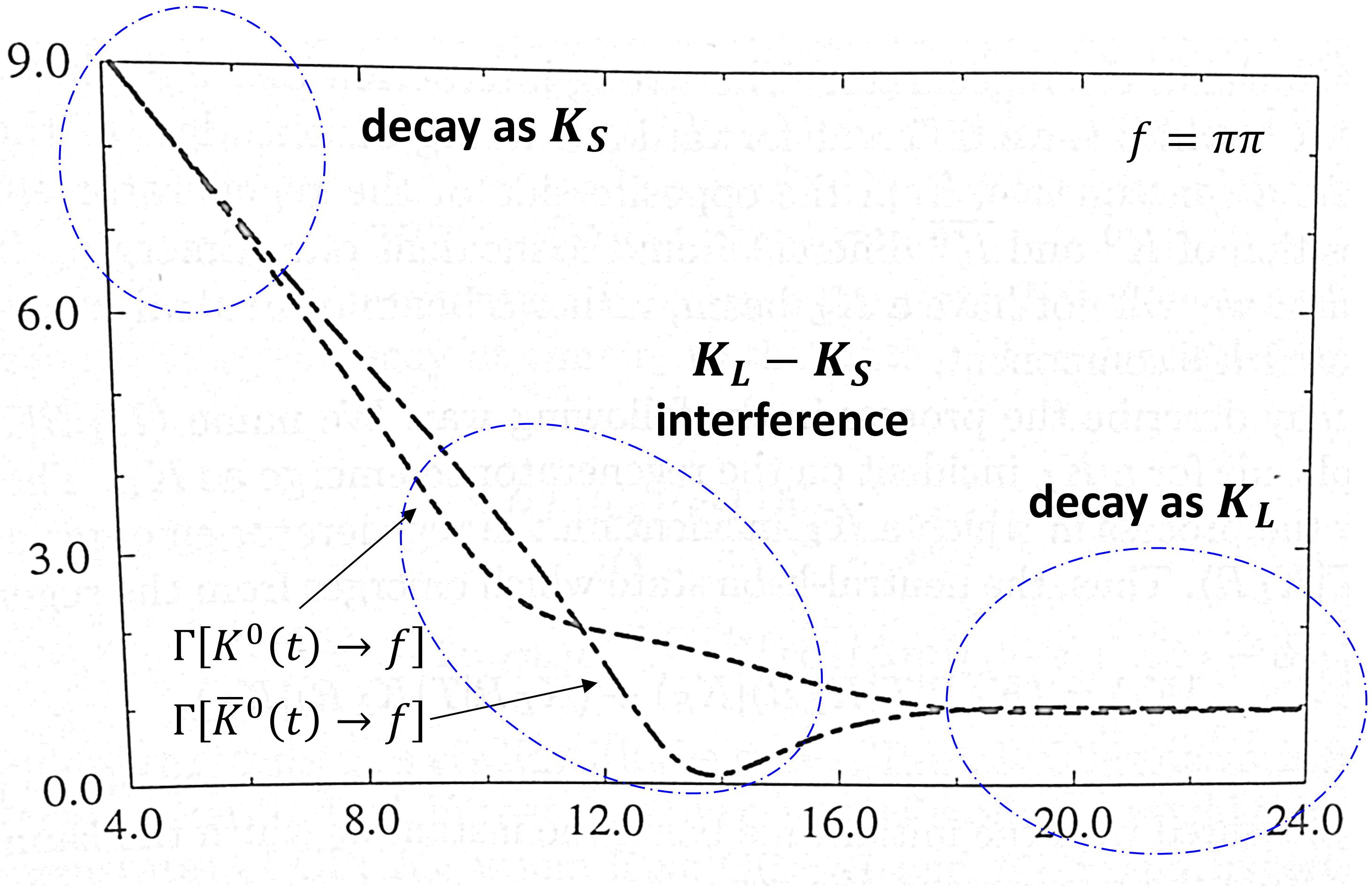
- Time evolution in general from pure $|K^0\rangle$ (f not necessarily CP eigenstate, CP violation allowed):

$$\Gamma[K^0(t) \to f] = \frac{1}{2}\left|A_f\right|^2 e^{-\Gamma t}(H + I) = \frac{|\langle f|\mathcal{H}|K_S\rangle|^2}{2(1-\delta_L)}\left[e^{-\Gamma_S t} + \left|\eta_f\right|^2 e^{-\Gamma_L t} + 2\left|\eta_f\right| e^{-\Gamma t}\cos\left(\phi_f - \Delta m\, t\right)\right]$$

physical phase

$$\eta_f \equiv \frac{\langle f|\mathcal{H}|K_L\rangle}{\langle f|\mathcal{H}|K_S\rangle} = \left|\eta_f\right| e^{i\phi_f} = \frac{1+\lambda_f}{1-\lambda_f} \to \left|\eta_f\right|^2 = \frac{\Gamma_L \mathrm{BR}(K_L \to f)}{\Gamma_S \mathrm{BR}(K_S \to f)} \qquad \delta_L \equiv |p|^2 - |q|^2 \quad \text{(CP-violating parameter)}$$

Neutral kaon time evolution



$$\tau_S = \frac{1}{\Gamma_S} \approx 9.0\times 10^{-11} s$$

$$\tau_L = \frac{1}{\Gamma_L} \approx 5.2\times 10^{-8} s$$

$$\Delta m = 0.53\times 10^{10}\ s^{-1}$$

$$\delta_L = 3.3\times 10^{-3}$$

$$|\eta_f| = \delta/\sqrt{2}$$

$$\phi_f = 43.5°$$

$|\langle f|\mathcal{H}|K_S\rangle|^2 = 10^6$ scaling factor

Neutral kaon decay modes: K_S

	Mode	Branching ratio (exp.)	Branching ratio (theo)
Non leptonic	$K_S \to \pi^0\pi^0$	$(30.69 \pm 0.05)\%$	
	$\pi^+\pi^-$	$(69.20 \pm 0.05)\%$	
	$\pi^+\pi^-\pi^0$	$(3.5^{+1.1}_{-0.9})\times10^{-7}$	
	$\pi^0\pi^0\pi^0$	$< 2.6\times10^{-8}$	1.9×10^{-9}
Non leptonic with photons	$\pi^+\pi^-\gamma$	$(1.79 \pm 0.05)\times10^{-3}$	
	$\pi^0\gamma\gamma$	$(4.9 \pm 1.8)\times10^{-8}$	
	$\gamma\gamma$	$(2.63 \pm 0.17)\times10^{-6}$	
Semileptonic	$\pi^\pm e^\mp \nu_e$	$(7.04 \pm 0.08)\times10^{-4}$	
	$\pi^\pm \mu^\mp \nu_\mu$	$-$	$\sim4.7\times10^{-4}$
Other rare	$\pi^0 e^+e^-$	$\left(3.0^{+1.5}_{-1.2}\right)\times10^{-9}$	
	$\pi^0\mu^+\mu^-$	$\left(2.9^{+1.5}_{-1.2}\right)\times10^{-9}$	
	$\mu^+\mu^-$	$< 2.1\times10^{-10}$	5.1×10^{-12}
	e^+e^-	$< 9\times10^{-9}$	2.1×10^{-14}

Neutral kaon decay modes: K_L (most relevant)

	Mode	Branching ratio (exp.)
Non leptonic	$K_L \to \pi^0\pi^0\pi^0$	$(19.52 \pm 0.12)\%$
	$\pi^+\pi^-\pi^0$	$(12.54 \pm 0.05)\%$
	$\pi^+\pi^-$	$(1.967 \pm 0.010)\times 10^{-3}$
	$\pi^0\pi^0$	$(0.864 \pm 0.006)\times 10^{-3}$
Semi-leptonic	$\pi^\pm e^\mp \nu_e$	$(40.55 \pm 0.11)\%$
	$\pi^\pm \mu^\mp \nu_\mu$	$(27.04 \pm 0.07)\%$
	$\pi^0\pi^\pm e^\mp \nu_e$	$(5.20 \pm 0.11)\times 10^{-5}$
Non leptonic with γ	$\pi^+\pi^-\gamma$	$(4.15 \pm 0.15)\times 10^{-5}$
	$\pi^0\gamma\gamma$	$(1.273 \pm 0.033)\times 10^{-6}$
	$\pi^0\gamma e^+e^-$	$(1.62 \pm 0.18)\times 10^{-8}$
	$\gamma\gamma$	$(5.47 \pm 0.04)\times 10^{-4}$

	Mode	Branching ratio (exp.)
Semileptonic with γ	$K_L \to e^+e^-\gamma$	$(9.4 \pm 0.4)\times 10^{-6}$
	$\mu^+\mu^-\gamma$	$(3.59 \pm 0.11)\times 10^{-7}$
Rare	$\mu^+\mu^-$	$(6.84 \pm 0.11)\times 10^{-9}$
	e^+e^-	$(9^{+6}_{-4})\times 10^{-12}$
	$\pi^0 e^+e^-$	$< 3.8\times 10^{-10}$
	$\pi^0\mu^+\mu^-$	$< 2.8\times 10^{-10}$
	$\pi^0\nu\bar{\nu}$	$< 3\times 10^{-9}$

Neutral kaons and *CP* violation

$K \to \pi\pi$ decays

- $K^0 \to \pi^+\pi^-$, $K^0 \to \pi^0\pi^0$
- $\pi\pi$ final states superposition of $|I, I_3\rangle = |0{,}0\rangle, |2{,}0\rangle$ (not $I = 1$ for Bose statistics)
- Amplitudes of K^0 to 2π states with total isospin I: $A_0 \equiv \langle 0{,}0|\mathcal{H}|K^0\rangle$, $A_2 \equiv \langle 2{,}0|\mathcal{H}|K^0\rangle$
- Isospin decomposition*: $A(K^0 \to \pi^+\pi^-) = \frac{1}{\sqrt{3}}\left(A_2 + \sqrt{2}A_0\right)$ and $A(K^0 \to \pi^0\pi^0) = \frac{1}{\sqrt{3}}\left(\sqrt{2}A_2 - A_0\right)$

strong and weak phases

CP conserving amplitudes

- It is possible to show that $A_{0,2}$ must be single-phase amplitudes (assuming CPT symmetry)

$$A_0 \equiv \langle 0|\mathcal{H}|K^0\rangle = a_0 e^{i(\delta_0+\phi_0)}$$

$$A_2 \equiv \langle 2|\mathcal{H}|K^0\rangle = a_2 e^{i(\delta_2+\phi_2)}$$

$a_0 \sim 20 a_2$ experimental (and theoretical) fact

$\Rightarrow$ $|A_0| = |\bar{A}_0|$, $|A_2| = |\bar{A}_2|$

* *Valid in the isospin limit (quarks d and u have equal masses)*

Neutral kaons and *CP* violation

$K \to \pi\pi$ decays

- Experimentally K^0 decay as K_S or K_L

- CP violating quantities (here $|0\rangle$ and $|2\rangle$ mean $|0,0\rangle$ and $|2,0\rangle$):

$$\varepsilon \equiv \frac{\langle 0|\mathcal{H}|K_L\rangle}{\langle 0|\mathcal{H}|K_S\rangle} \Rightarrow \varepsilon = \frac{pA_0 + q\bar{A}_0}{pA_0 - q\bar{A}_0} = \frac{1+\lambda_0}{1-\lambda_0}$$

Because $|0\rangle$ is a *CP* eigenstate, $|\lambda_0| \neq 1$ and $\mathcal{J}m(\lambda_0) \neq 0$ denote *CP* violation in mixing and interference, respectively

$\varepsilon \neq 0$ implies *CP* violation either in mixing and/or interference

$$\varepsilon' \equiv \frac{\langle 2|\mathcal{H}|K_L\rangle\langle 0|\mathcal{H}|K_S\rangle - \langle 0|\mathcal{H}|K_L\rangle\langle 2|\mathcal{H}|K_S\rangle}{\sqrt{2}\langle 0|\mathcal{H}|K_S\rangle^2} \propto A_0\bar{A}_2 - A_2\bar{A}_0 = 2ia_0a_2e^{i(\delta_0+\delta_2)}\sin(\phi_0-\phi_2)$$

$\varepsilon' \neq 0$ implies direct *CP* violation

Neutral kaons and *CP* violation

- Assuming $\langle 2|\mathcal{H}|K_S\rangle \ll \langle 0|\mathcal{H}|K_S\rangle$

$$\varepsilon \approx \frac{2\eta_{+-} + \eta_{00}}{3} \qquad \varepsilon' \approx \frac{\eta_{+-} - \eta_{00}}{3}$$

with

$$\eta_{+-} = \frac{\langle \pi^+\pi^-|\mathcal{H}|K_L\rangle}{\langle \pi^+\pi^-|\mathcal{H}|K_S\rangle} = |\eta_{+-}|e^{i\phi_{+-}} \qquad \eta_{00} = \frac{\langle \pi^0\pi^0|\mathcal{H}|K_L\rangle}{\langle \pi^0\pi^0|\mathcal{H}|K_S\rangle} = |\eta_{00}|e^{i\phi_{00}}$$

- $|\eta_{+-,00}|$ and $\phi_{+-,00}$ are observables measured
  - from the proper time evolution of the 2π decay rate
  - from the branching ratio of $K_{L,S}$ to 2π and $\tau_{L,S}$ ($|\eta_{+-,00}|$)
  - From regeneration effect*

- PDG fit to all the measurements to date (NA31, CPLEAR, NA48, KTeV, KLOE)

$$|\eta_{+-}| = (2.232 \pm 0.011)\times 10^{-3} \qquad \phi_{+-} = (43.5 \pm 0.5)^\circ$$

$$|\eta_{00}| = (2.220 \pm 0.011)\times 10^{-3} \qquad \phi_{00} = (43.7 \pm 0.6)^\circ$$

($\phi_{+-} = \phi_{00}$ from CPT invariance)

* *Not discussed in these lectures*

Neutral kaons and *CP* violation in mixing

- Measurement of $|\varepsilon|$ from the measurements of $\left|\eta_{+-,00}\right|$

$$|\varepsilon| \approx \frac{2|\eta_{+-}| + |\eta_{00}|}{3}$$

(valid because $\phi_{+-} \approx \phi_{00}$ and $\mathcal{R}e(\varepsilon'/\varepsilon) \ll 1$ see later)

- Measurement of ϕ_ε from

$$\phi_\varepsilon \approx \frac{2\phi_{+-} + \phi_{00}}{3}$$

(valid because $\phi_{+-} \approx \phi_{00}$ and $\mathcal{R}e(\varepsilon'/\varepsilon) \ll 1$ see later)

Regeneration effect (KTeV)

- PDG fit

$|\varepsilon| = (2.228 \pm 0.011)\times 10^{-3}$ $\phi_\varepsilon = (43.5 \pm 0.5)°$

ϵ prediction in the Standard Model

- Relation between ε and δ

$$\left.\begin{array}{l} \delta_L \equiv |p|^2 - |q|^2 \rightarrow \lambda_0 \equiv \dfrac{q}{p}\dfrac{\bar{A}_0}{A_0} = \sqrt{\dfrac{1-\delta}{1+\delta}}\,e^{i\theta} \\ \varepsilon = \dfrac{1+\lambda_0}{1-\lambda_0}, \qquad \text{and } \varepsilon = |\varepsilon| e^{i\phi_\varepsilon} \end{array}\right\} \quad \delta_L \approx 2\mathcal{R}e(\varepsilon) = 2|\varepsilon|\cos\phi_\varepsilon = (3.232 \pm 0.016)\times 10^{-3}$$

- δ predictable in SM

  Using the formalism of the neutral meson oscillation and the relation valid for kaons $\Gamma_{12} \approx A_0^* \bar{A}_0$ is possible to demonstrate

$$\delta \approx -\frac{\mathcal{J}m(M_{12} A_0 \bar{A}_0^*)}{\Delta m\,(A_0 \bar{A}_0^*)}$$

  - The amplitude $A_0 \propto V_{us}^* V_{ud}$ (tree-level) and the hadronic matrix element simplify in the ratio
  - M_{12} is the dispersive amplitude of the box diagram
  - Δm is measured

The neutral B system

- $B^0 - \bar{B}^0$: theory and experiments → $|q/p| \approx 1$ (no CP violation in mixing)
- Final states
  - Almost the same for B^0 and $\bar{B}^0$
  - CP-eigenstates with both eigenvalues, or non CP eigenstates
  - Large variety of phase space availability

- **Small CP violation in mixing** → $|P_L\rangle$ and $|P_H\rangle$ ~ CP eigenstates with eigenvalues +1 and -1
- Almost **no difference** between P_H and P_L decay (experimental and theoretical fact)
- $|P_L\rangle$ and $|P_H\rangle$
  - $\Delta\Gamma = \Gamma_L - \Gamma_H \approx 0$
  - We can speak in terms of B^0 ($\bar{B}^0$) decays.

The neutral B system: time-dependent asymmetries

- $B^0 - \bar{B}^0$: theory and experiments $\rightarrow |q/p| \approx 1$ (no CP violation in mixing), $\Delta\Gamma \approx 0$
- If f is a final states common to both B^0 and $\bar{B}^0$

$$\Gamma(B^0(t) \rightarrow f) = \frac{1}{2}\left|A_f\right|^2 e^{-\Gamma t}(H + I)$$
$$\Gamma(\bar{B}^0(t) \rightarrow f) = \frac{1}{2}\left|A_f\right|^2 e^{-\Gamma t}(H - I)$$

where $H = 1 + \left|\lambda_f\right|^2$, $\quad I = \left(1 - \left|\lambda_f\right|^2\right)\cos(\Delta m\, t) + 2\mathcal{Im}\left(\lambda_f\right)\sin(\Delta m\, t)$

- **Time-dependent asymmetry**

$$\mathcal{A}(t) \equiv \frac{\Gamma(\bar{B}^0(t) \rightarrow f) - \Gamma(B^0(t) \rightarrow f)}{\Gamma(\bar{B}^0(t) \rightarrow f) + \Gamma(B^0(t) \rightarrow f)} = S_f \sin(\Delta m_B t) - C_f \cos(\Delta m_B t)$$

$$S_f = \frac{2\mathcal{Im}\left(\lambda_f\right)}{1 + \left|\lambda_f\right|^2}$$

$$C_f = \frac{1 - \left|\lambda_f\right|^2}{1 + \left|\lambda_f\right|^2}$$

- If f is a ***CP* eigenstate**, $\mathcal{A}(t)$ **violates *CP***: $S_f \neq 0$ **interference** CP-violation, $C_f \neq 0$ **direct** CP-violation

Measurement of the CKM angle $\boldsymbol{\gamma}$

- Weak phase between $b \to c$ (Cabibbo—favoured "fav") and $b \to u$(Cabibbo – suppressed "sup") quark transitions

$$\boldsymbol{\gamma} = \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$$

- **Measurement**
  - processes receiving contributions from both Cabibbo-favoured and suppressed amplitudes
  - study the interference between the two amplitudes

- **Gronau-London-Wyler method (GLW)**
  - Build observables of "fav"-"sup" amplitudes interference from the decays

    $B^+ \to \overline{D}^0 K^+ \qquad B^+ \to D^0 K^+ \qquad B^- \to \overline{D}^0 K^- \qquad B^- \to D^0 K^- \qquad B^\pm \to D_{CP} K^\pm$
  - $D^0, \overline{D}^0$ = flavour-specific D final state (e.g. (~) $D^0 \to K^-\pi^+$, $\overline{D}^0 \to K^+\pi^-$)
  - D_{CP} = CP-eigenstate D final state (e.g. $\pi^+\pi^-, K^+K^-, K_S\pi^0$, ...)

  N.B. no need to study time-dependent asymmetries (charged B mesons)

Measurement of the CKM angle γ

- Only 1 tree-level amplitude in "fav" and "sup"

  $|A_{+\bar{D}}| = |A_{-D}| \equiv |A_{fav}|$

  $|A_{+D}| = |A_{-\bar{D}}| \equiv |A_{sup}|$

- CP - conservation in strong/em interactions

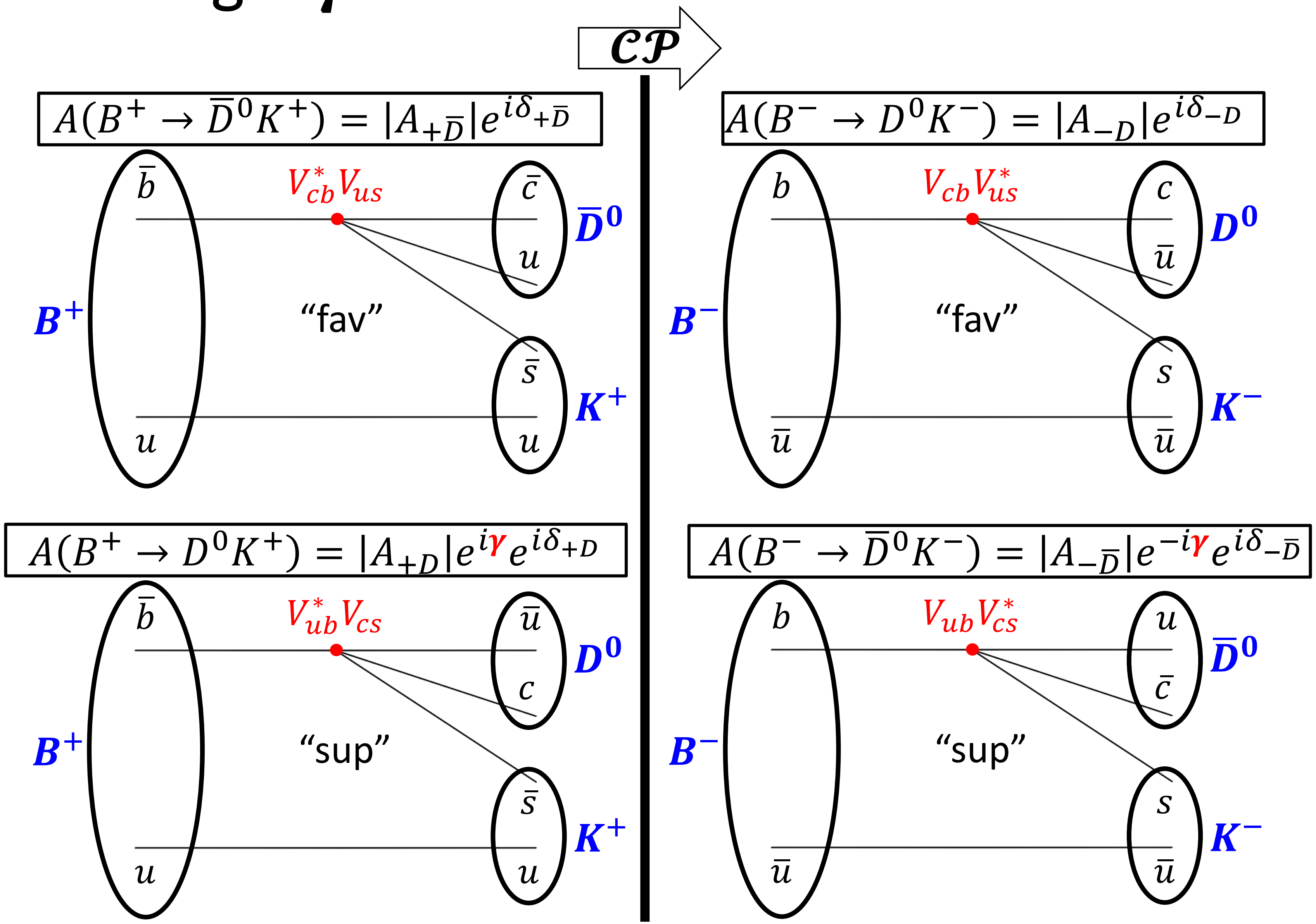
  $\delta_{+\bar{D}} = \delta_{-D} \equiv \delta_f$

  $\delta_{+D} = \delta_{-\bar{D}} \equiv \delta_s$

- Notation

  $r_B \equiv |A_{sup}|/|A_{fav}|$

  $\delta_B \equiv \delta_s - \delta_f$



Measurement of the CKM angle $\boldsymbol{\gamma}$

$\boxed{\boldsymbol{B^{\pm} \to D_{CP} K^{\pm}}}$

- hyp. 1: no D^0 oscillation
- hyp. 2: no CP violation in D decay

$$\Rightarrow \quad A(B^+ \to D_{CP\pm} K^+) = \frac{1}{\sqrt{2}} [A(B^+ \to D^0 K^+) \pm A(B^+ \to \bar{D}^0 K^+)]$$

$$A(B^- \to D_{CP\pm} K^-) = \frac{1}{\sqrt{2}} [A(B^- \to D^0 K^-) \pm A(B^- \to \bar{D}^0 K^-)]$$

Observables

$$R_{CP\pm} \equiv 2 \frac{\Gamma(B^- \to D_{CP\pm} K^-) + \Gamma(B^+ \to D_{CP\pm} K^+)}{\Gamma(B^- \to D^0 K^-) + \Gamma(B^+ \to \bar{D}^0 K^+)} = 1 + r_B^2 \pm 2 r_B \cos \boldsymbol{\delta_B} \cos \boldsymbol{\gamma}$$

$$\mathcal{A}_{CP\pm} \equiv \frac{\Gamma(B^- \to D_{CP\pm} K^-) - \Gamma(B^+ \to D_{CP\pm} K^+)}{\Gamma(B^- \to D_{CP\pm} K^-) + \Gamma(B^+ \to D_{CP\pm} K^+)} = \frac{\pm 2 r_B \sin \boldsymbol{\delta_B} \sin \boldsymbol{\gamma}}{1 + r_B^2 \pm 2 r_B \cos \boldsymbol{\delta_B} \cos \boldsymbol{\gamma}}$$

$$r_B = \frac{\Gamma(B^- \to \bar{D}^0 K^-)}{\Gamma(B^- \to D^0 K^-)} = \frac{\Gamma(B^+ \to D^0 K^+)}{\Gamma(B^+ \to \bar{D}^0 K^+)}$$

Measurement of the CKM angle γ

- **Gronau-London-Wyler method** (GLW)
  - Experimental difficulty due to small r_B leading to large uncertainty
  - Angular solution up to a four – fold ambiguity
  - D^0 oscillation cannot be fully neglected
- **GLW example (BaBar)**
  - Measure R_{CP+}, R_{CP-}, $\mathcal{A}_{CP+}$, $\mathcal{A}_{CP-}$
  - Extract the parameters γ,δ_B,r_B
  - Decays: $B^{\pm} \to D h^{\pm}$ with $h = K, \pi$

D_{CP+}	$[K^+K^-]_D h^{\pm}$	$[\pi^+\pi^-]_D h^{\pm}$	
D_{CP-}	$[K_S\pi^0]_D h^{\pm}$	$[K_S\phi]_D h^{\pm}$	$[K_S\omega]_D h^{\pm}$
Non - CP	$[K^-\pi^+]_{D^0} h^-$	$[K^+\pi^-]_{\bar{D}^0} h^+$	

$(K_S \to \pi^+\pi^-, \phi \to K^+K^-, \omega \to \pi^+\pi^-\pi^0)$

Measurement of the CKM angle γ

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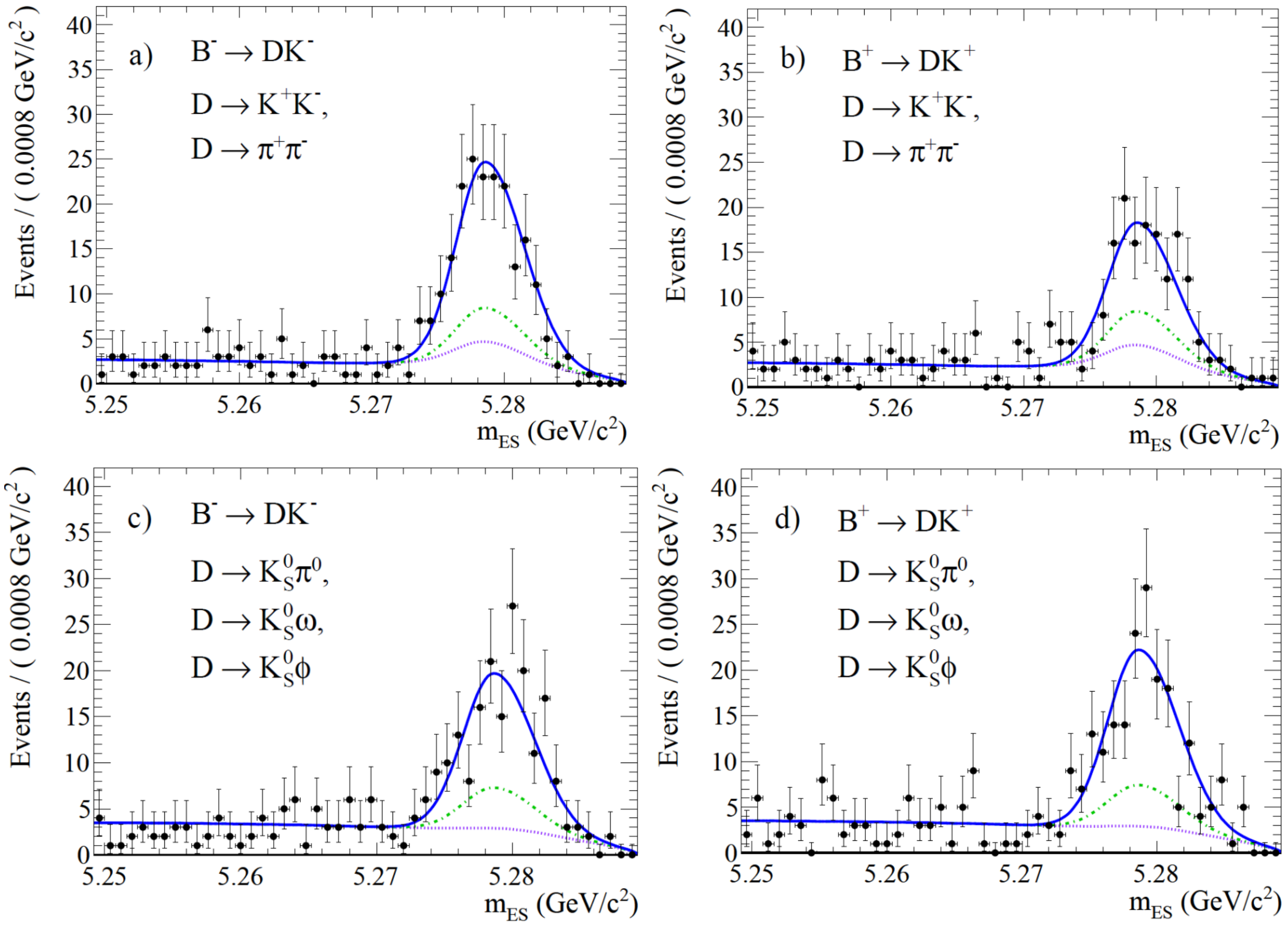
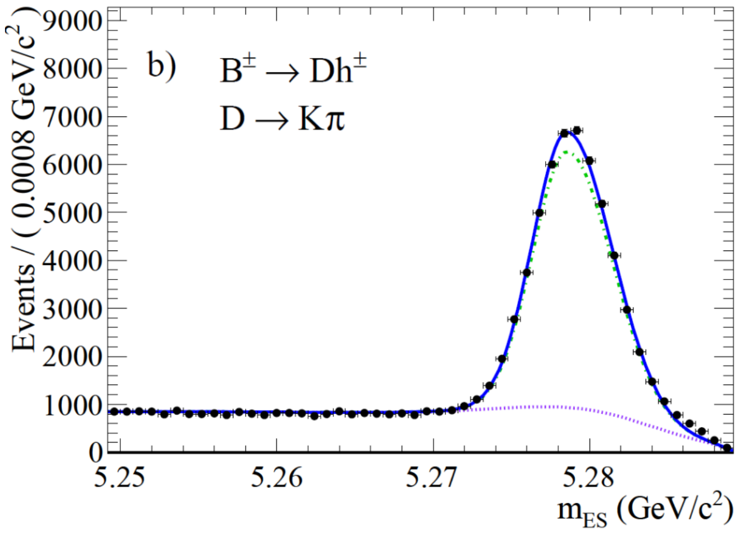
- **GLW example (BaBar)**

D^0 mode	$N(B^{\pm} \to DK^{\pm})$	$N(B^{\pm} \to D\pi^{\pm})$
K^+K^-	367 ± 27	4091 ± 70
$\pi^+\pi^-$	110 ± 9	1230 ± 41
$K_S^0\pi^0$	338 ± 24	4182 ± 73
$K_S^0\omega$	116 ± 9	1440 ± 45
$K_S^0\phi$	52 ± 4	648 ± 27
$K^-\pi^+$	3361 ± 82	44631 ± 232





$$m_{\mathrm{ES}} = \sqrt{(s/2 + \mathbf{p}_{ee} \cdot \mathbf{p}_B)^2 / E_{ee}^2 - p_B^2},$$

Measurement of the CKM angle γ

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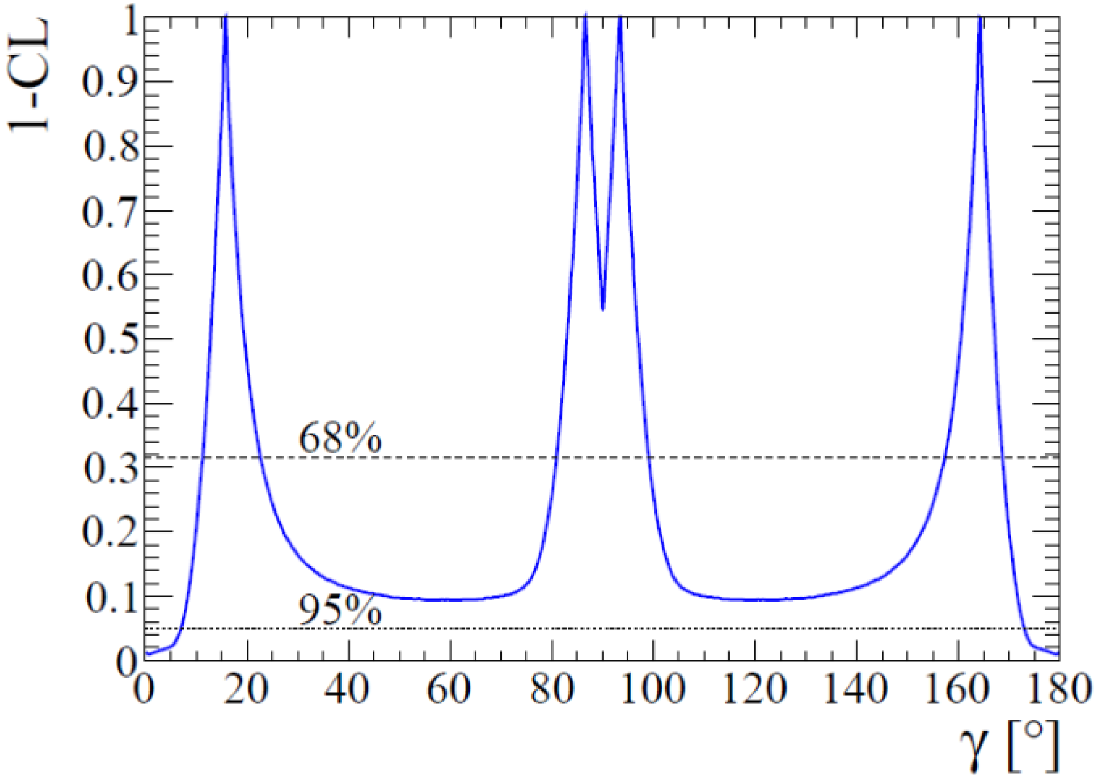
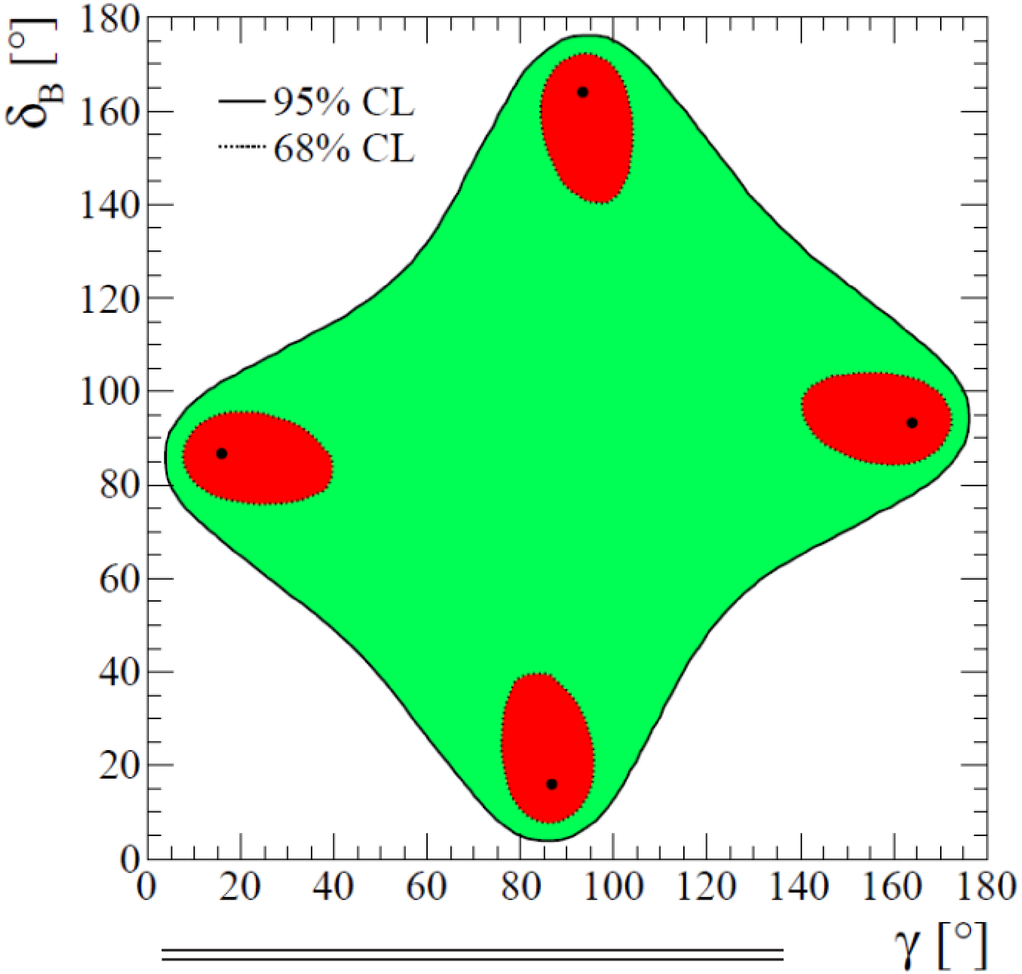
- **GLW example (BaBar)**

$$A_{CP+} = 0.25 \pm 0.06(\mathrm{stat}) \pm 0.02(\mathrm{syst}),$$
$$A_{CP-} = -0.09 \pm 0.07(\mathrm{stat}) \pm 0.02(\mathrm{syst}),$$
$$R_{CP+} = 1.18 \pm 0.09(\mathrm{stat}) \pm 0.05(\mathrm{syst}),$$
$$R_{CP-} = 1.07 \pm 0.08(\mathrm{stat}) \pm 0.04(\mathrm{syst}).$$



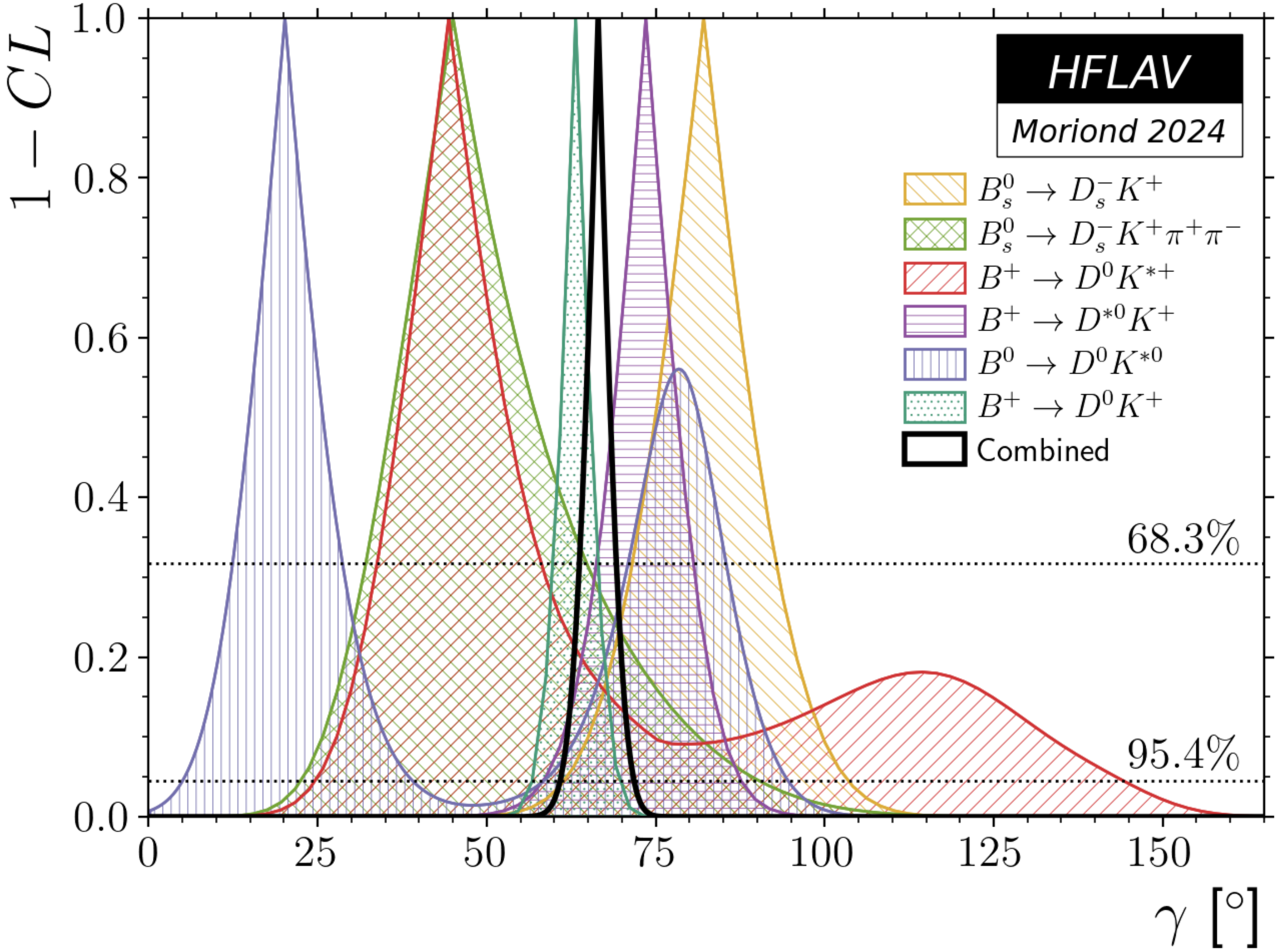


	γ mod 180 [°]
68% CL	[11.3, 22.7] [80.8, 99.2] [157.3, 168.7]
95% CL	[7.0, 173.0]

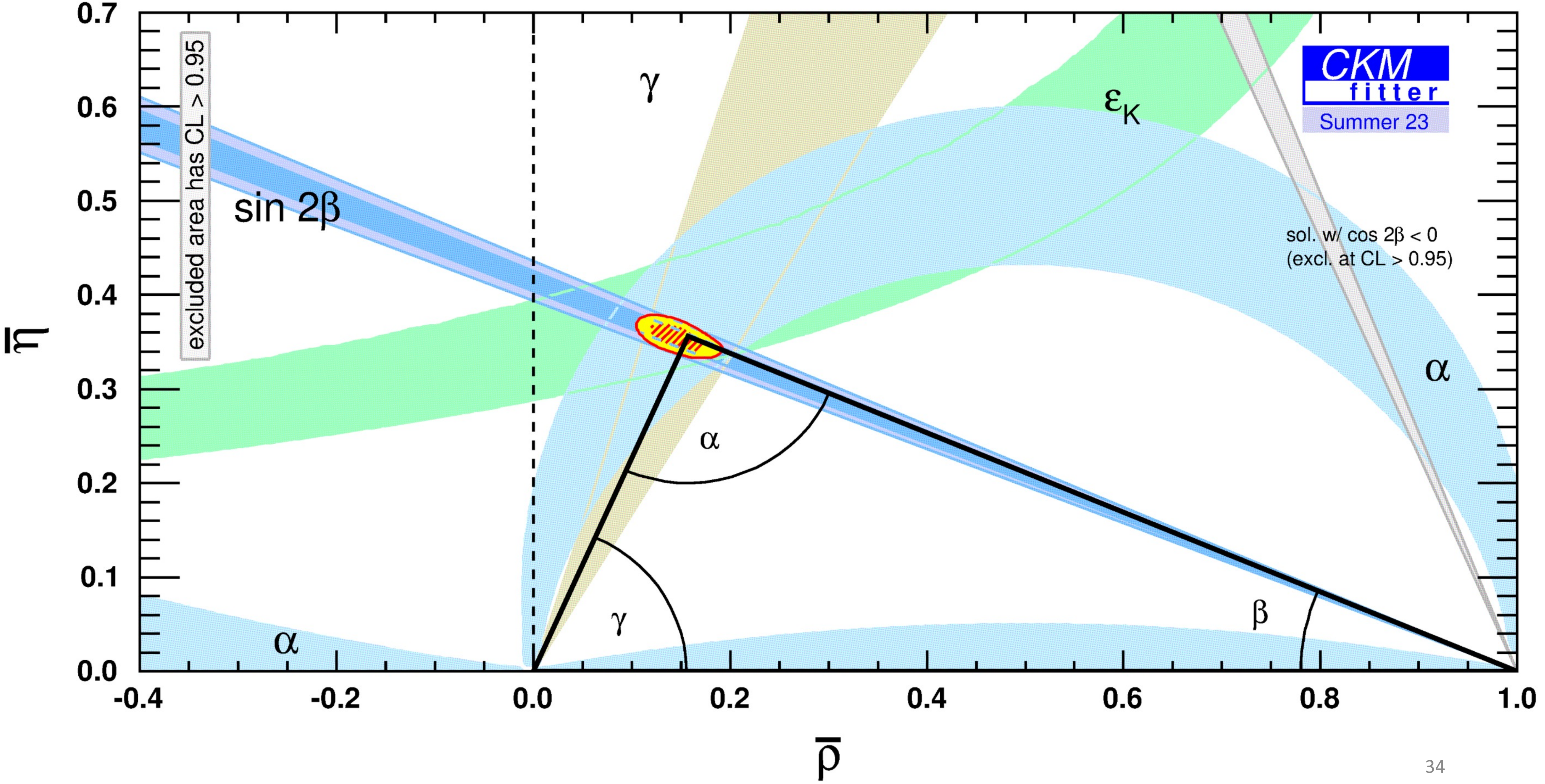
Measurement of the CKM angle γ: combination

- Other methods are analysis of multibody decays of D mesons (Dalitz analysis)
- Combinations of the results make use of all the possible ratios from various types of methods

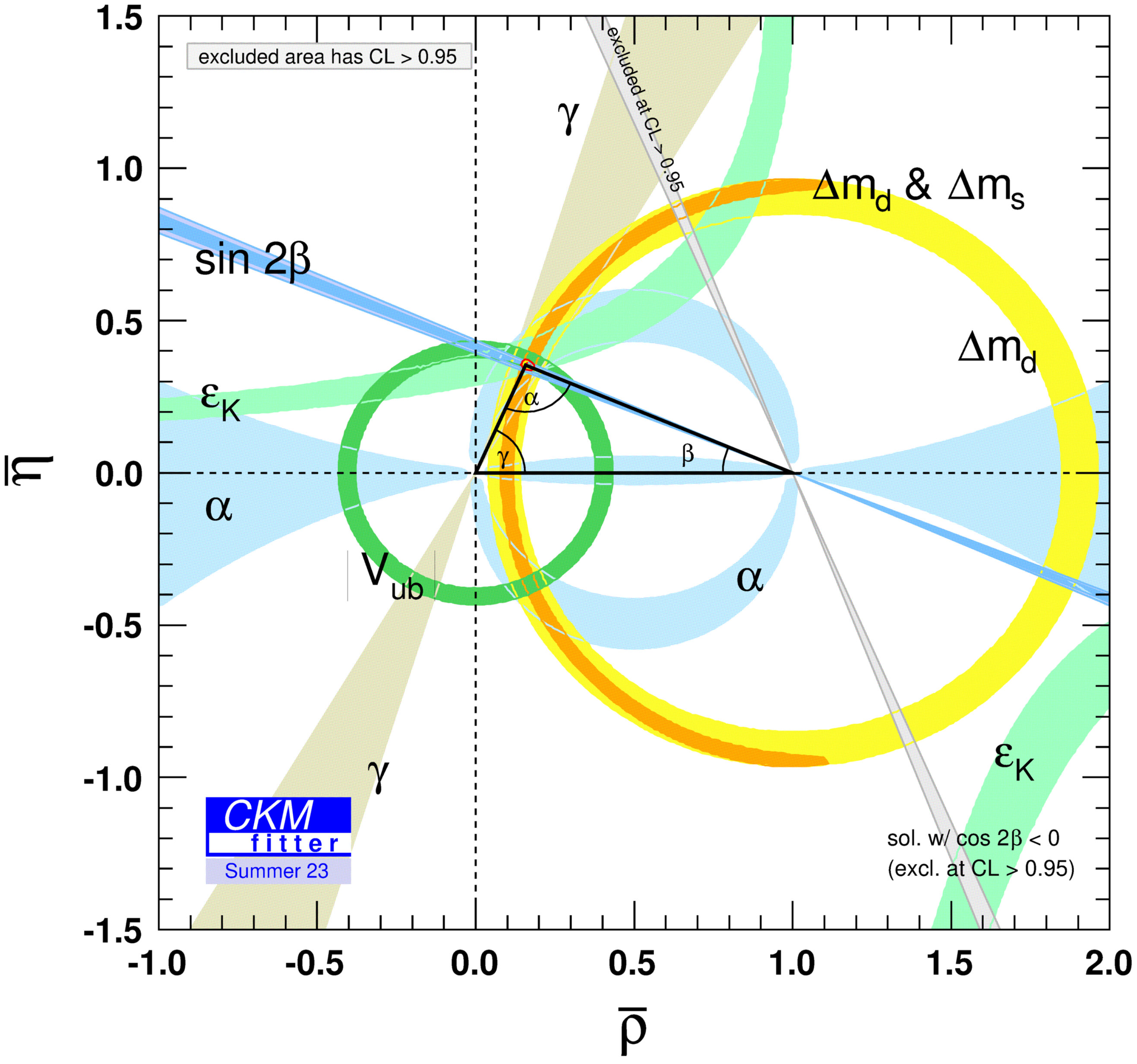
$$\gamma = \left(65.9^{+3.3}_{-3.5}\right)^{\circ}$$



The CKM angle γ and the unitarity triangle



CKM fit (CKMFitter)



$$A = 0.8215^{+0.0047}_{-0.0082}$$

$$\lambda = 0.22498^{+0.00023}_{-0.00021}$$

$$\bar{\rho} = 0.1562^{+0.0112}_{-0.0040}$$

$$\bar{\eta} = 0.3551^{+0.0051}_{-0.0057}$$

Wolfenstein parameters

$$J = \left(3.115^{+0.047}_{-0.059}\right)\times 10^{-5}$$

Jarlskog invariant

68% CL

Summary of Lecture 13

Main learning outcomes

- What are the different types of CP-violation in the Standard Model
- Phenomenology of CP-violation in charged and neutral meson decays (kaons, B mesons)
- Experimental measurements of CP-violation in the different meson systems